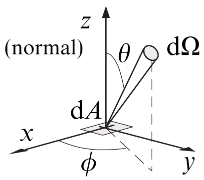
5.7 Radiation processes

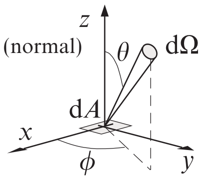
Radiometry[a]

Radiant energy[b]	$Q_e = \iiint L_e \cos\theta \, dA \, d\Omega \, dt \quad \mathrm{J}$	(5.145)	Q_e radiant energy L_e radiance (generally a function of position and direction) θ angle between dir. of $d\Omega$ and normal to dA Ω solid angle
Radiant flux ("radiant power")	$\Phi_e = \frac{\partial Q_e}{\partial t} \quad \mathrm{W}$	(5.146)	A area t time Φ_e radiant flux
	$= \iint L_e \cos\theta \, dA \, d\Omega$	(5.147)	
Radiant energy density[c]	$W_e = \frac{\partial Q_e}{\partial V} \quad \mathrm{J\,m^{-3}}$	(5.148)	W_e radiant energy density dV differential volume of propagation medium
Radiant exitance[d]	$M_e = \frac{\partial \Phi_e}{\partial A} \quad \mathrm{W\,m^{-2}}$	(5.149)	M_e radiant exitance
	$= \int L_e \cos\theta \, d\Omega$	(5.150)	
Irradiance[e]	$E_e = \frac{\partial \Phi_e}{\partial A} \quad \mathrm{W\,m^{-2}}$	(5.151)	
	$= \int L_e \cos\theta \, d\Omega$	(5.152)	
Radiant intensity	$I_e = \frac{\partial \Phi_e}{\partial \Omega} \quad \mathrm{W\,sr^{-1}}$	(5.153)	E_e irradiance I_e radiant intensity
	$= \int L_e \cos\theta \, dA$	(5.154)	
Radiance	$L_e = \frac{1}{\cos\theta} \frac{\partial^2 \Phi_e}{dA \, d\Omega} \quad \mathrm{W\,m^{-2}\,sr^{-1}}$	(5.155)	
	$= \frac{1}{\cos\theta} \frac{\partial I_e}{\partial A}$	(5.156)	



[a]Radiometry is concerned with the treatment of light as energy.
[b]Sometimes called "total energy." Note that we assume opaque radiant surfaces, so that $0 \le \theta \le \pi/2$.
[c]The instantaneous amount of radiant energy contained in a unit volume of propagation medium.
[d]Power per unit area leaving a surface. For a perfectly diffusing surface, $M_e = \pi L_e$.
[e]Power per unit area incident on a surface.

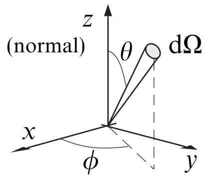
Photometry[a]

Luminous energy ("total light")	$Q_v = \iiint L_v \cos\theta \, dA \, d\Omega \, dt$ lm s	(5.157)	Q_v luminous energy L_v luminance (generally a function of position and direction) θ angle between dir. of $d\Omega$ and normal to dA Ω solid angle
Luminous flux	$\Phi_v = \frac{\partial Q_v}{\partial t}$ lumen (lm) $= \iint L_v \cos\theta \, dA \, d\Omega$	(5.158) (5.159)	A area t time Φ_v luminous flux
Luminous density[b]	$W_v = \frac{\partial Q_v}{\partial V}$ $\mathrm{lm\,s\,m^{-3}}$	(5.160)	W_v luminous density V volume
Luminous exitance[c]	$M_v = \frac{\partial \Phi_v}{\partial A}$ lx $(\mathrm{lm\,m^{-2}})$ $= \int L_v \cos\theta \, d\Omega$	(5.161) (5.162)	M_v luminous exitance 
Illuminance ("illumination")[d]	$E_v = \frac{\partial \Phi_v}{\partial A}$ $\mathrm{lm\,m^{-2}}$ $= \int L_v \cos\theta \, d\Omega$	(5.163) (5.164)	
Luminous intensity[e]	$I_v = \frac{\partial \Phi_v}{\partial \Omega}$ cd $= \int L_v \cos\theta \, dA$	(5.165) (5.166)	E_v illuminance I_v luminous intensity
Luminance ("photometric brightness")	$L_v = \frac{1}{\cos\theta} \frac{\partial^2 \Phi_v}{dA \, d\Omega}$ $\mathrm{cd\,m^{-2}}$ $= \frac{1}{\cos\theta} \frac{\partial I_v}{\partial A}$	(5.167) (5.168)	
Luminous efficacy	$K = \frac{\Phi_v}{\Phi_e} = \frac{L_v}{L_e} = \frac{I_v}{I_e}$ $\mathrm{lm\,W^{-1}}$	(5.169)	K luminous efficacy L_e radiance Φ_e radiant flux I_e radiant intensity
Luminous efficiency	$V(\lambda) = \frac{K(\lambda)}{K_{max}}$	(5.170)	V luminous efficiency λ wavelength K_{max} spectral maximum of $K(\lambda)$

[a] Photometry is concerned with the treatment of light as seen by the human eye.
[b] The instantaneous amount of luminous energy contained in a unit volume of propagating medium.
[c] Luminous emitted flux per unit area.
[d] Luminous incident flux per unit area. The derived SI unit is the lux (lx). $1\,\mathrm{lx} = 1\,\mathrm{lm\,m^{-2}}$.
[e] The SI unit of luminous intensity is the candela (cd). $1\,\mathrm{cd} = 1\,\mathrm{lm\,sr^{-1}}$.

Radiative transfer[a]

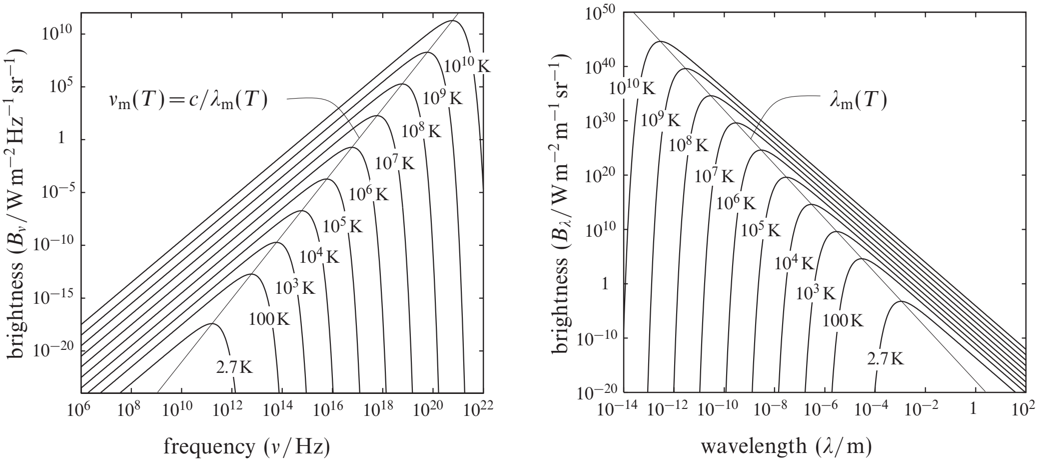
Quantity	Equation	Units	No.
Flux density (through a plane)	$F_\nu = \int I_\nu(\theta,\phi)\cos\theta \, \mathrm{d}\Omega$	$\mathrm{W\,m^{-2}\,Hz^{-1}}$	(5.171)
Mean intensity[b]	$J_\nu = \frac{1}{4\pi}\int I_\nu(\theta,\phi)\, \mathrm{d}\Omega$	$\mathrm{W\,m^{-2}\,Hz^{-1}}$	(5.172)
Spectral energy density[c]	$u_\nu = \frac{1}{c}\int I_\nu(\theta,\phi)\, \mathrm{d}\Omega$	$\mathrm{J\,m^{-3}\,Hz^{-1}}$	(5.173)
Specific emission coefficient	$j_\nu = \frac{\epsilon_\nu}{\rho}$	$\mathrm{W\,kg^{-1}\,Hz^{-1}\,sr^{-1}}$	(5.174)
Gas linear absorption coefficient ($\alpha_\nu \ll 1$)	$\alpha_\nu = n\sigma_\nu = \frac{1}{l_\nu}$	$\mathrm{m^{-1}}$	(5.175)
Opacity[d]	$\kappa_\nu = \frac{\alpha_\nu}{\rho}$	$\mathrm{kg^{-1}\,m^2}$	(5.176)
Optical depth	$\tau_\nu = \int \kappa_\nu \rho \, \mathrm{d}s$		(5.177)
Transfer equation[e]	$\frac{1}{\rho}\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu + j_\nu$		(5.178)
	or $\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\alpha_\nu I_\nu + \epsilon_\nu$		(5.179)
Kirchhoff's law[f]	$S_\nu \equiv \frac{j_\nu}{\kappa_\nu} = \frac{\epsilon_\nu}{\alpha_\nu}$		(5.180)
Emission from a homogeneous medium	$I_\nu = S_\nu(1-\mathrm{e}^{-\tau_\nu})$		(5.181)



F_ν	flux density
I_ν	specific intensity ($\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$)
J_ν	mean intensity
u_ν	spectral energy density
Ω	solid angle
θ	angle between normal and direction of Ω
j_ν	specific emission coefficient
ϵ_ν	emission coefficient ($\mathrm{W\,m^{-3}\,Hz^{-1}\,sr^{-1}}$)
ρ	density
α_ν	linear absorption coefficient
n	particle number density
σ_ν	particle cross section
l_ν	mean free path
κ_ν	opacity
τ_ν	optical depth, or optical thickness
$\mathrm{d}s$	line element
S_ν	source function

[a]The definitions of these quantities vary in the literature. Those presented here are common in meteorology and astrophysics. Note particularly that the ambiguous term *specific* is taken to mean "per unit frequency interval" in the case of specific intensity and "per unit mass per unit frequency interval" in the case of specific emission coefficient.
[b]In radio astronomy, flux density is usually taken as $S = 4\pi J_\nu$.
[c]Assuming a refractive index of 1.
[d]Or "mass absorption coefficient."
[e]Or "Schwarzschild's equation."
[f]Under conditions of local thermal equilibrium (LTE), the source function, S_ν, equals the Planck function, $B_\nu(T)$ [see Equation (5.182)].

Blackbody radiation



Planck function[a]	$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]^{-1} \quad (5.182)$$ $$B_\lambda(T) = B_\nu(T)\frac{d\nu}{d\lambda} \quad (5.183)$$ $$= \frac{2hc^2}{\lambda^5}\left[\exp\left(\frac{hc}{\lambda kT}\right)-1\right]^{-1} \quad (5.184)$$	B_ν surface brightness per unit frequency (W m^{-2} Hz^{-1} sr^{-1}) B_λ surface brightness per unit wavelength (W m^{-2} m^{-1} sr^{-1}) h Planck constant
Spectral energy density	$$u_\nu(T) = \frac{4\pi}{c}B_\nu(T) \quad \mathrm{J\,m^{-3}\,Hz^{-1}} \quad (5.185)$$ $$u_\lambda(T) = \frac{4\pi}{c}B_\lambda(T) \quad \mathrm{J\,m^{-3}\,m^{-1}} \quad (5.186)$$	c speed of light k Boltzmann constant T temperature $u_{\nu,\lambda}$ spectral energy density
Rayleigh–Jeans law ($h\nu \ll kT$)	$$B_\nu(T) = \frac{2kT}{c^2}\nu^2 = \frac{2kT}{\lambda^2} \quad (5.187)$$	
Wien's law ($h\nu \gg kT$)	$$B_\nu(T) = \frac{2h\nu^3}{c^2}\exp\left(\frac{-h\nu}{kT}\right) \quad (5.188)$$	
Wien's displacement law	$$\lambda_{\rm m}T = \begin{cases} 5.1\times10^{-3}\,\mathrm{m\,K} & \text{for } B_\nu \\ 2.9\times10^{-3}\,\mathrm{m\,K} & \text{for } B_\lambda \end{cases} \quad (5.189)$$	$\lambda_{\rm m}$ wavelength of maximum brightness
Stefan–Boltzmann law[b]	$$M = \pi\int_0^\infty B_\nu(T)\,d\nu \quad (5.190)$$ $$= \frac{2\pi^5k^4}{15c^2h^3}T^4 = \sigma T^4 \quad \mathrm{W\,m^{-2}} \quad (5.191)$$	M exitance σ Stefan–Boltzmann constant ($\simeq 5.67\times10^{-8}$ W m^{-2} K^{-4})
Energy density	$$u(T) = \frac{4}{c}\sigma T^4 \quad \mathrm{J\,m^{-3}} \quad (5.192)$$	u energy density
Greybody	$$M = \epsilon\sigma T^4 = (1-A)\sigma T^4 \quad (5.193)$$	ϵ mean emissivity A albedo

[a]With respect to the projected area of the surface. Surface brightness is also known simply as "brightness." "Specific intensity" is used for reception.

[b]Sometimes "Stefan's law." Exitance is the total radiated energy from unit area of the body per unit time.